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 (April 28, 2000)

It is shown that bipartition of optical solitons can be used to generate entangled light beams. The achievable amount of entanglement can be substantially larger for N -bound solitons ($N = 2, 3$) than for the fundamental soliton ($N = 1$). An analysis of the mode structure of the entangled beams shows that just N modes are essentially entangled. In particular, partitioning of the fundamental soliton effectively produces 2-mode squeezed light.

Quantum coherent dynamics of atomic and photonic systems has been a subject of increasing interest stimulated by the rapidly developing field of quantum information processing and quantum computing [1–5]. Intrinsic parallelism of quantum state evolution renders it possible to overcome the problem of exponential time required by a classical computer for solving complex problems such as integer factorization [6,7], discrete logarithm [6], and search [8]. High sensitivity of quantum coherent dynamics to eavesdropping can be used for developing secure encryption protocols [9] realized in fiber optical systems [10].

One of the most remarkable features of quantum coherence is entanglement, on which, e.g., quantum teleportation is based [11]. Entanglement can be regarded as being the nonclassical contribution to the overall correlation between two parts of a system (see, e.g., [12,13]). Typically, discrete systems that are built up by qubits have been considered. Concepts that use continuous quantum variables have recently attracted increasing publicity [14–19]. An illustrative example is the teleportation of coherent states by means of entangled light of the squeezed-vacuum type [16].

A way to generate entangled light is to use two fields each of which is prepared in some nonclassical state and combine them at a linear four-port device like a beam splitter or a coupler. In particular, the entangled squeezed light used in [16] is produced by combining two light beams independently squeezed via $\chi^{(2)}$ parametric processes. The possibility of realizing entangled pulse sources by combining two separately squeezed optical solitons are discussed in [20]. Alternatively, a nonlinear coupling between two fields can prepare them in an entangled (continuous-variable) state. For example, the $\chi^{(2)}$ process of parametric down-conversion has been extensively used as a source of entangled beams and pulses (see, e.g., [21,22]). In this letter we show that bipartition of (fundamental and N -bound) optical solitons formed in a medium with a Kerr-type $\chi^{(3)}$ nonlinearity yields entangled light fields, because of the internal quantum correlations of the solitons.

From classical optics it is well known that the Kerr nonlinearity can compensate for dispersion-assisted pulse

spreading or diffraction-assisted beam broadening (see, e.g., [23,24]). In both cases, the evolution of the (complex) field amplitude $a(x, t)$ can be described by the nonlinear Schrödinger equation (NSE)

$$i\partial_t a(x, t) = -\frac{1}{2}\omega^{(2)}\partial_{xx}a(x, t) + \chi|a(x, t)|^2a(x, t), \quad (1)$$

(t , propagation variable; x , “transverse” coordinate; $\omega^{(2)}$, second-order dispersion or diffraction constant; χ , nonlinear-coupling constant). Bright temporal solitons can be formed either in focusing media with anomalous dispersion ($\chi < 0$, $\omega^{(2)} > 0$) or in defocusing media with normal dispersion ($\chi > 0$, $\omega^{(2)} < 0$). Spatial solitons always require a focusing nonlinearity. Instead of working in x -space, we can also turn to the ω -space,

$$\underline{a}(\omega, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx a(x, t) e^{i\omega x}. \quad (2)$$

In what follows the coordinates in the x - and ω -domains are scaled by the initial soliton width x_0 and the spectral width $\omega_0 = 1/x_0$ respectively. Propagation distances are measured in dispersion (diffraction) lengths $t_d = x_0^2/|\omega^{(2)}|$. The NSE (1) can be solved by the inverse scattering method [25]. In particular, the bell-shaped fundamental soliton is extremely stable and can propagate over distances far beyond the limit given by the dispersion- (diffraction-) assisted spreading. Scaling the power of the fundamental soliton by a factor of N^2 , $N = 2, 3, \dots$, yields so-called N -bound solitons [26]. The two classes of solutions of the NSE have been realized experimentally (see, e.g., [27–29]).

In quantum theory, the complex field amplitude $a(x, t)$ and $a^*(x, t)$ become field operator $\hat{a}(x, t)$ and $\hat{a}^\dagger(x, t)$, respectively, with

$$[\hat{a}(x, t), \hat{a}^\dagger(x', t)] = \delta(x - x'), \quad (3)$$

and the NSE becomes an operator-valued equation [30,31]. In order to solve the quantum mechanical problem, it is commonly assumed that the initial state is a multimode coherent state such that $\langle \hat{a} \rangle \sim N \text{sech}(x/x_0)$ is the initial shape of the classical N -bound-soliton solution. For not too large propagation distances the state

then evolves into a multimode squeezed state of Gaussian type, which can be calculated numerically within the framework of appropriate discretization [32].

The nonlinear soliton motion leads to internal quantum correlations, which have been studied both experimentally [33] and theoretically [32,34–36]). From the results the question is suggested of whether or not it is possible to produce two entangled light beams by appropriate partitioning of quantum solitons. Let us consider bipartition as sketched in Fig. 1, which may be realized experimentally using spectral filtering [37,38] or spatial filtering [31]. Since the soliton quantum state remains pure during the propagation, the entanglement E of the bipartite system is given by the von Neumann entropy of a subsystem [12] i.e.,

$$E = S_1 = -\text{Tr}_1(\hat{\rho}_1 \ln \hat{\rho}_1) = S_2 = -\text{Tr}_2(\hat{\rho}_2 \ln \hat{\rho}_2), \quad (4)$$

where $\hat{\rho}_i$ is the density operator of the i th subsystem, and Tr_i means the trace with regard to the i th subsystem ($i = 1, 2$).

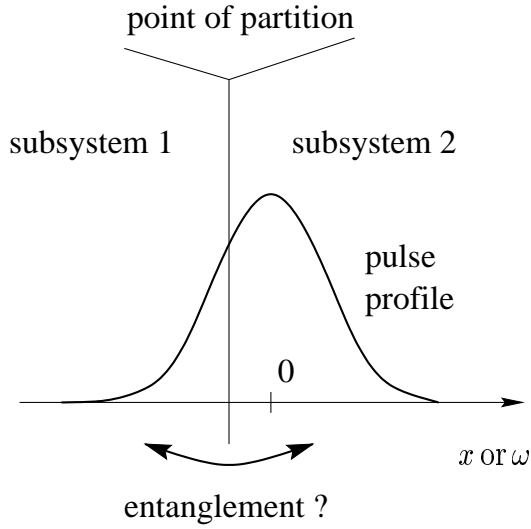


FIG. 1. Bipartition of optical solitons. Applying appropriate filtering techniques (see, e.g., [31,37,38]), it can be realized in the x -domain by partition of the near field of spatial solitons and in the ω -domain by partition of the far field of spatial solitons or spectral partition of temporal solitons.

In order to calculate the entropy of a subsystem prepared in a Gaussian state, let us consider the x -domain and assume that the subsystem extends over a region Ξ (the calculation in the ω -domain is analogous). We introduce new bosonic operators

$$\hat{b}_k = \int_{\Xi} dx [\mu_k(x) \Delta \hat{a}(x) + \nu_k(x) \Delta \hat{a}^\dagger(x)] \quad (5)$$

$[\Delta \hat{a}(x) = \hat{a}(x) - \langle \hat{a}(x) \rangle]$ and impose on the $\mu_k(x)$ and $\nu_k(x)$ the condition that

$$\langle \hat{b}_k^\dagger \hat{b}_{k'} \rangle = \sigma_k \delta_{kk'}, \quad \langle \hat{b}_k \hat{b}_{k'} \rangle = 0. \quad (6)$$

For notational convenience, we have omitted the argument t . From Eqs. (5) and (6) it then follows that the parameters σ_k and the functions $\mu_k(x)$ and $\nu_k(x)$ solve the equations

$$\begin{aligned} \begin{bmatrix} (\sigma_k + \frac{1}{2}) \delta_{kk'} \\ 0 \end{bmatrix} &= \int_{\Xi} dx \int_{\Xi} dx' \begin{bmatrix} \mu_k(x) & \nu_k(x) \\ \nu_k^*(x) & \mu_k^*(x) \end{bmatrix} \\ &\times \begin{bmatrix} C(x, x')^* & B(x, x') \\ B(x, x')^* & C(x, x') \end{bmatrix} \begin{bmatrix} \mu_{k'}^*(x') \\ \nu_{k'}^*(x') \end{bmatrix}, \end{aligned} \quad (7)$$

where

$$B(x, x') = \langle \Delta \hat{a}(x) \Delta \hat{a}(x') \rangle, \quad (8)$$

$$C(x, x') = \langle \Delta \hat{a}^\dagger(x) \Delta \hat{a}(x') \rangle + \frac{1}{2} \delta(x - x'). \quad (9)$$

Note that the functions $\mu_k(x)$ and $\nu_k(x)$ must satisfy the conditions

$$\int_{\Xi} dx [\mu_k(x) \mu_{k'}^*(x) - \nu_k(x) \nu_{k'}^*(x)] = \delta_{kk'}, \quad (10)$$

$$\int_{\Xi} dx [\mu_k(x) \nu_{k'}(x) - \nu_k(x) \mu_{k'}(x)] = 0, \quad (11)$$

because of the commutation relations $[\hat{b}_k, \hat{b}_{k'}^\dagger] = \delta_{kk'}$ and $[\hat{b}_k, \hat{b}_{k'}] = 0$. With regard to the new bosonic operators, the Gaussian state (of the subsystem) is obviously a multimode thermal state, with σ_k being the mean photon number of the k th mode. Hence, the entanglement E is given by

$$E = \sum_k S_{\text{th}}(\sigma_k), \quad (12)$$

with

$$S_{\text{th}}(\sigma) = \ln \left[\frac{(\sigma + 1)^{\sigma+1}}{\sigma^\sigma} \right] \quad (13)$$

being the entropy of a single-mode thermal state. Note that when the Ξ -region comprises the whole soliton prepared in a pure Gaussian state, then all the σ_k vanish and thus $E = 0$, as expected.

Figure 2 presents examples of the entanglement obtained by symmetrical bipartition of the fundamental soliton and the 2- and 3-bound solitons in both the x -domain [solid lines in Fig. 2(b)] and the ω -domain [solid lines in Fig. 2(c)] as a function of the propagation distance. The figure clearly shows that highly entangled parties can be realized by bipartition of optical solitons produced in Kerr-like media. The (numerical) calculations have shown that the maximum entanglement is typically observed for symmetrical bipartition (i.e., at $x = 0$ or $\omega = 0$). For the system considered in the figure, an asymmetrical bipartition of the 3-bound soliton in the x -domain at $|x| \lesssim 0.4 x_0$ can improve the entanglement by 5.3% for certain propagation distances.

The entanglement realized by bipartition of the fundamental soliton increases monotonously with the propagation distance. The maxima and minima of the entanglement realized by bipartition in the x -domain of 2- and 3-bound solitons follow quite exactly the periodic change between soliton compression and expansion [compare Figs. 2(a) and 2(b)]. In the ω -domain this correlation is less pronounced [compare Figs. 2(a) and 2(c)]. It is worth noting that the amount of entanglement that can be achieved increases with the order parameter N .

Formally, the sum in Eq. (12) runs over all modes. In fact, only a few modes substantially contribute to the entanglement, as it is seen from Figures 2 and 3 (the modes are numbered such that the single-mode contribution to the entanglement decreases with increasing mode index). In particular, it is seen that the number of relevant modes is just given by the soliton-order parameter N . A possible reason for this fact can be seen in the multicomponent structure of the classical N -soliton solution as discussed in [36].

In summary, the results offer novel possibilities of using optical solitons as sources of entangled light beams. Although bipartition of solitons again yields multimode objects, only a few modes are involved in the entanglement. In particular, fundamental solitons are suited for generation of entangled light like 2-mode squeezed light, which is an alternative to the standard $\chi^{(2)}$ sources.

ACKNOWLEDGMENTS

This work was supported by the Deutsche Forschungsgemeinschaft. We are grateful to Tomaš Opatrný for valuable discussions.

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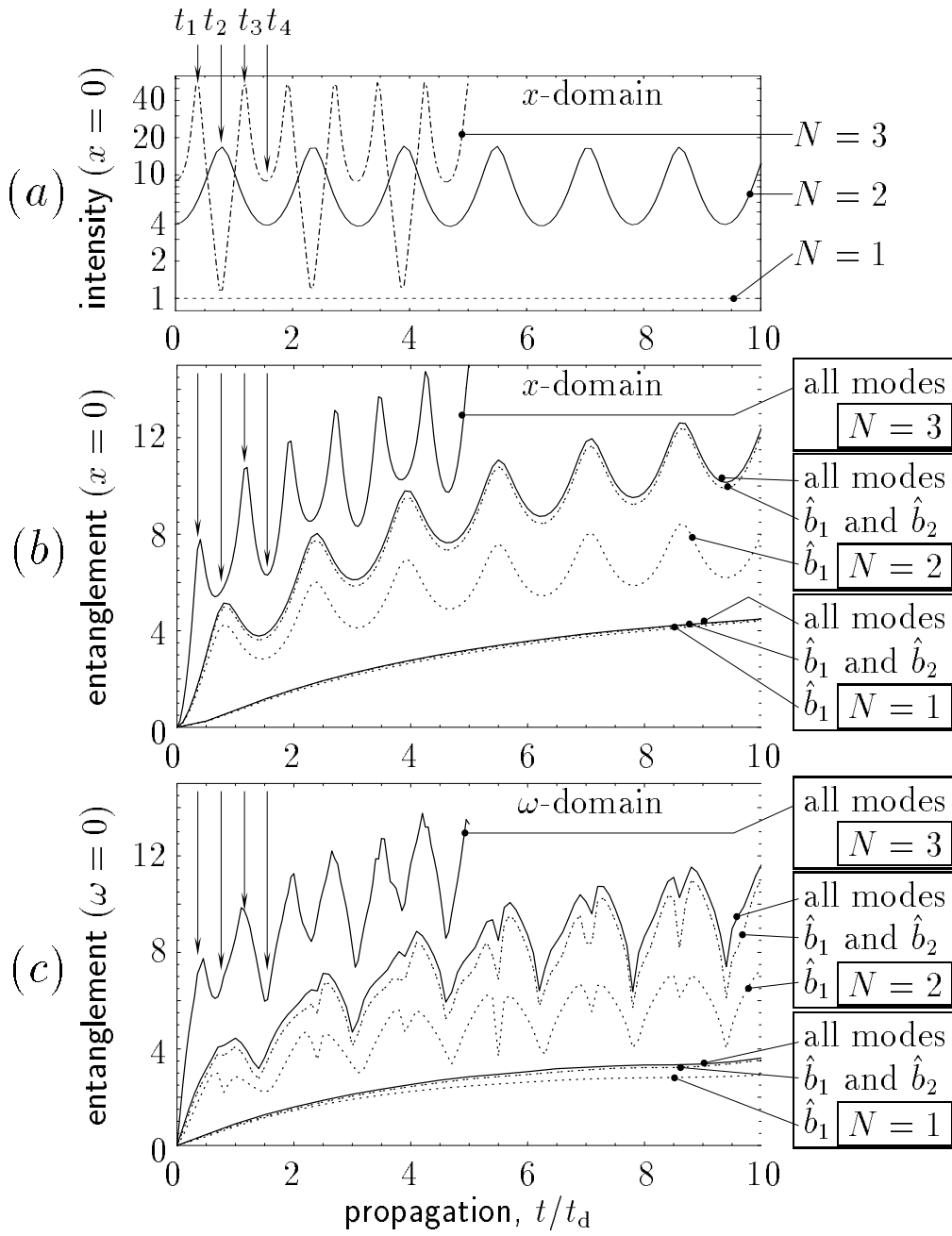


FIG. 2. Dependence on the propagation distance of the entanglement realized by symmetrical bipartition of the fundamental soliton ($N = 1$) and N -bound solitons ($N = 2, 3$) in the x -domain (b) and the ω -domain (c). For comparison, the soliton mid-intensity in the x -domain as a function of the propagation distance is shown (a). The contributions to the entanglement of the relevant modes are shown for the fundamental soliton and the 2-bound soliton. The marked propagation distances are: $t_4 = \frac{\pi}{2} t_d$ (soliton period), $t_2 = \frac{1}{2} t_4$ (distance of compression for 2-bound soliton), $t_1 = \frac{1}{4} t_4$ and $t_3 = \frac{3}{4} t_4$ (distances of compression for 3-bound soliton). In the numerical calculations, which are performed on a grid of 256 points with a discretization step $\Delta x = 0.05 x_0$, it is assumed that the total photon numbers of the solitons are $2N^2 \bar{n}$, where $\bar{n} = 10^9$ (for details, see [31,32]).

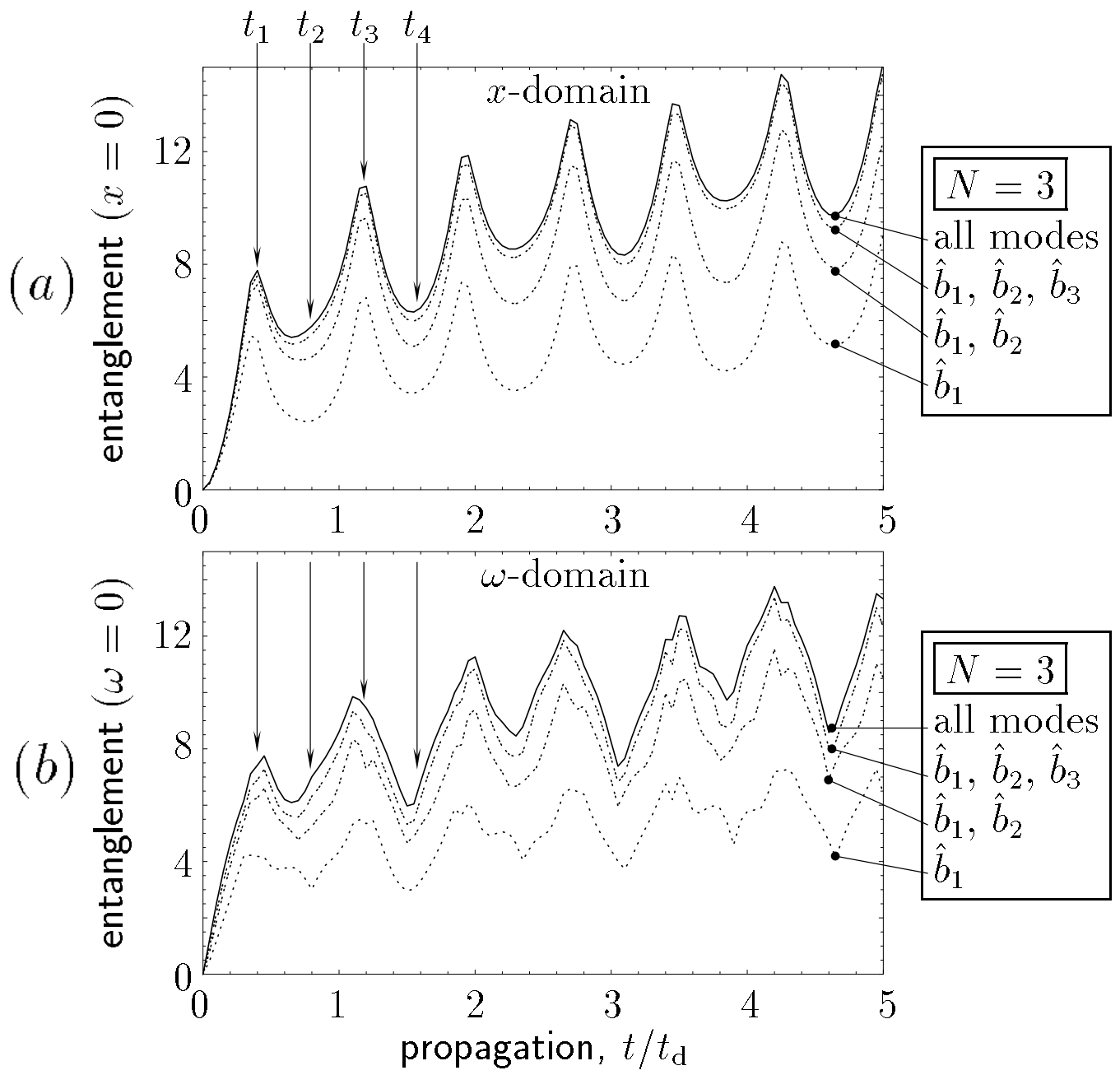


FIG. 3. The contributions of the relevant modes to the entanglement realized by symmetrical bipartition of the 3-bound soliton in the x -domain (a) and the ω -domain (b). The soliton is the same as in Fig. 2.

